Example

Determine the strength of the Sun's gravitational field at the position of the Earth. Justify any assumptions.

mass of Sun = 2.0×10^{30} kg

distance to the Sun from Earth = 1.5×10^{11} m

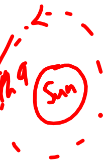
$$g = \frac{GM}{r^2}$$

5.9×10^{-3} N kg^{-1}

Use this information to determine:

a) the centripetal acceleration of the Earth in its orbit about the Sun. 5.9×10^{-3} m s^{-2}

$F_{net} = ma$



b) the speed of the Earth in its orbit

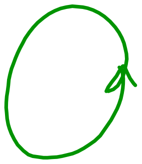
$m_{earth} g = m_{earth} a$

$a = g$

c) the period of the Earth's orbit

b)
$$a = \frac{v^2}{r}$$
$$v^2 = ar$$
$$v^2 = (5.9 \times 10^{-3}\ \text{m s}^{-2})(1.5 \times 10^{11}\ \text{m})$$
$$v = 3.0 \times 10^{4}\ \text{m s}^{-1}$$

c)
$$v = \frac{\Delta d}{\Delta t}$$
$$v = \frac{2\pi r}{T}$$
$$T = \frac{2\pi r}{v}$$
$$T = \frac{2\pi(1.5 \times 10^{11}\ \text{m})}{3.0 \times 10^{4}\ \text{m s}^{-1}}$$



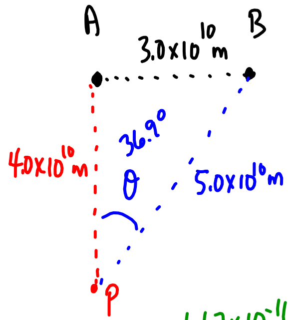
$2\pi r$

$$T = 3.2 \times 10^{7}\ \text{s}$$
$$T = 8778.8\ \text{h}$$
$$T = 365.8\ \text{d}$$

Example

A binary star system is composed of two stars A and B separated by a distance of 3.0×10^{10} m. The mass of star A is 1.6×10^{31} kg and the mass of star B is 2.5×10^{31} kg. Point P is 4.0×10^{10} m from Star A.

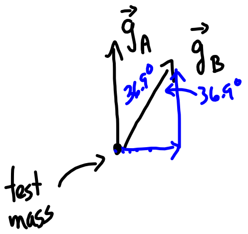
* Determine the gravitational field strength due to the two stars at P.



$\tan\theta = \frac{3.0}{4.0}$

$\theta = 36.9°$

Consider placing a test mass at P. Draw a FBD for the test mass:



$$g_A = \frac{G(1.6\times10^{31}\text{ kg})}{(4.0\times10^{10}\text{ m})^2}$$

$G = 6.67\times10^{-11}\text{ N m}^2\text{ kg}^{-2}$

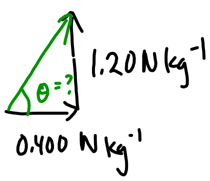
$$g_A = 0.667\text{ N kg}^{-1}$$

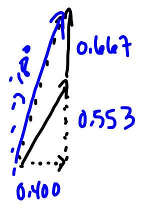
$$g_B = \frac{G(2.5\times10^{+31}\text{ kg})}{(5.0\times10^{10}\text{ m})^2}$$

$$g_B = 0.667\text{ N kg}^{-1}$$

	x	y
g_A	0	0.667 N kg^{-1}
g_B	0.667 sin36.9°	0.667 cos36.9°
g_{net}	0.400 N kg^{-1}	1.20 N kg^{-1}

OR Vector Addition Diagram





$$c^2 = (1.20)^2 + (0.400)^2$$

$$c = 1.3\text{ N kg}^{-1}$$

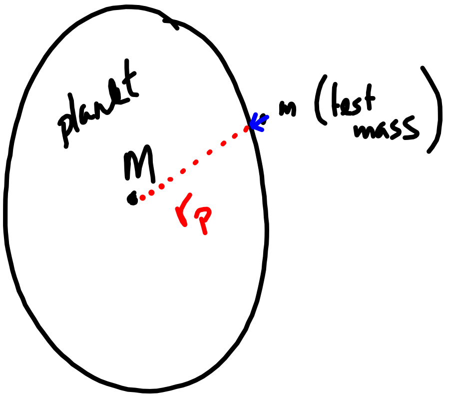
$$\tan\theta = \frac{1.20}{0.400}$$

$$\theta = 72°$$

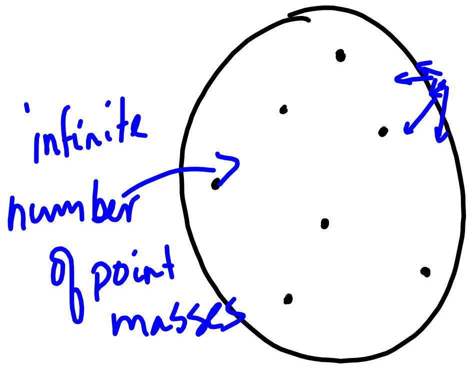
The field strength at P is 1.3 N kg^{-1} inclined at angle of 18° to the line joining P and A.

Gravitational Field Strength at the Surface of a planet

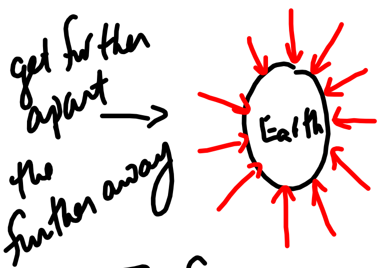
- assuming that all the mass is concentrated at its centre.



$$g = \frac{GM}{r_p^2}$$

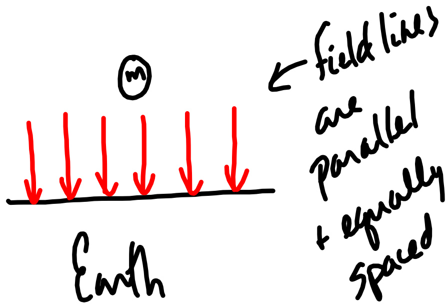


Gravitational field Pattern of the Earth:



The field is directed radially inward

(Big scale)



(smaller scale)

Example

Assuming that the Earth is a uniform sphere, use the data below to show that the gravitational field strength of the Earth at its surface is approximately $9.8\,N\,kg^{-1}$.

mass of Earth = 5.97×10^{24} kg

radius of Earth = 6.38×10^{6} m

$$g = \frac{GM}{r_p^2}$$

$$g = \frac{G(5.97 \times 10^{24}\,kg)}{(6.38 \times 10^{6}\,m)^2}$$

$$g = 9.78\,N\,kg^{-1} \approx 9.8\,N\,kg^{-1}$$

Determine the gravitational field strength at the international space station (altitude of 350 km)

$r = 350\,km + 6.38 \times 10^{6}\,m$

$r = 3.5 \times 10^{5}\,m + 6.38 \times 10^{6}\,m$

$r = 6.73 \times 10^{6}\,m$

$$g = \frac{G(5.97 \times 10^{24}\,kg)}{(6.73 \times 10^{6}\,m)^2}$$

$g = 8.79\,N\,kg^{-1}$

How high must an astronaut be before her weight is half the value it is at the Earth's surface?

$W = mg$

$g = \frac{9.78\,N\,kg^{-1}}{2}$

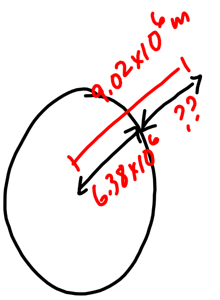
$g = 4.89\,N\,kg^{-1}$

$$g = \frac{GM}{r^2}$$

$$r^2 = \frac{GM}{g}$$

$$r^2 = \frac{G(5.97 \times 10^{24}\,kg)}{(4.89\,N\,kg^{-1})}$$

$r = 9.02 \times 10^{6}\,m$



$h = 9.02 \times 10^{6}\,m - 6.38 \times 10^{6}\,m$

$h = 2.64 \times 10^{6}\,m$

Example

$r = 3392 \text{ km}$

The mass of Mars is 6.43×10^{23} kg and its diameter is 6794 km

What is the weight of a 5.00 kg object on the surface of Mars?

① $W = mg \longrightarrow g = \frac{GM}{r^2}$

$W = (5.00 \text{ kg})(3.73 \text{ N kg}^{-1})$

$g = \frac{G(6.43 \times 10^{23} \text{ kg})}{(3392 \times 10^3 \text{ m})^2}$

$W = 18.6 \text{ N}$

$g = 3.73 \text{ N kg}^{-1}$

② Use $F = \frac{G m_1 m_2}{r^2}$ ← g

↑ Force of gravity due to m_1 or m_2

(weight due to m_1 or m_2)